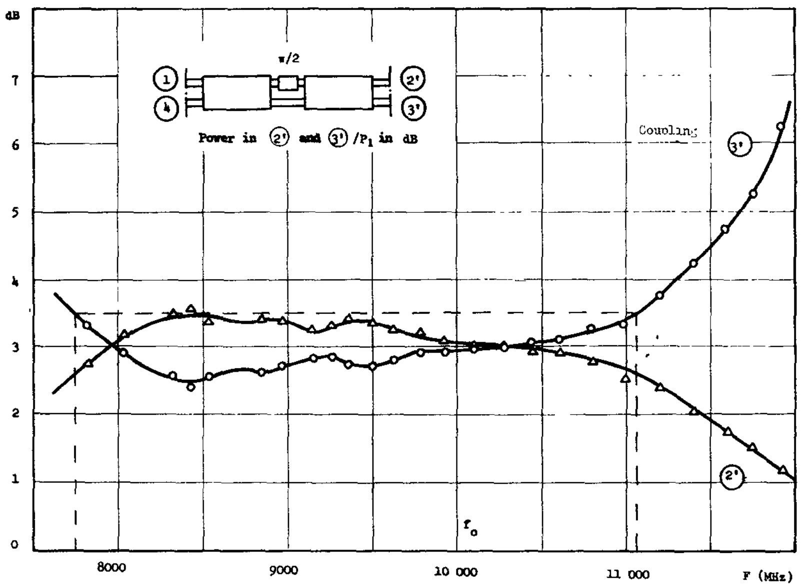

Fig. 4. Coupling of the whole hybrid versus frequency.

$a = 0.6$ mm and $c = 1.23$ mm. The characteristics of the fourteen-slot couplers used are shown in Fig. 2. The couplers used have a coupling higher than 2.6 dB over 2350 MHz (bandwidth of 23 percent). The VSWR measured at one port when all the others are matched is less than 1.08 between 8 and 12 GHz and the directivity is better than 30 dB. The quadrature phase shifter is constructed by inserting, in the waveguide (RG 52/U), a dielectric slab tapered at both ends to minimize reflections.

The dimensions of this slab have been calculated from the theory described by Halford [3] and Altmann [4].

The whole junction we built [5] is realized with two fourteen-slot couplers and one $\pi/2$ phase shifter designed as above. For this whole junction we measured the directivity, the coupling, and the VSWR at one port when all the others were matched. The results are shown in Fig. 3 and Fig. 4. The VSWR is less than 1.1 between 8.3 and 12 GHz. The most interesting property remains the decoupling between the input 1 and the output 4 when arms 2' and 3' are terminated by two identical impedances (here short circuits). This decoupling is higher than 40 dB for the two frequencies where the coupling of each coupler is exactly 3 dB. We have shown that a junction endowed with the magic tee properties over a range wider than 3 GHz in the X band could be obtained.

M. Bouthinon and A. Coumes
Ecole Nationale Supérieure
d'Electronique
University of Grenoble
Grenoble, France

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Normal Incidence on Semi-Infinite Longitudinally Drifting Magneto-Plasma: The Nonrelativistic Solution

The purpose of this correspondence is to present a nonrelativistic solution to the problem of normal incidence of electromagnetic waves on a semi-infinite longitudinally drifting homogeneous cold magnetoplasma. Reflected and transmitted waves from the drifting boundary are found and the results are identical with a relativistic solution presented by Chawla and Unz [1].

Let a linearly polarized plane electromagnetic wave in free space be normally incident in the positive z direction on the drifting boundary of a semi-infinite cold magnetoplasma which is drifting with a constant velocity $\bar{v}_0 = v_0\hat{z}$ with a superimposed static magnetic field $\bar{H}_0 = H_0\hat{z}$; the incident wave will be given by

$$\bar{E}_I = E_x^I \hat{x} e^{i(\omega_I t - k_I z)};$$
$$\bar{H}_I = \frac{1}{\eta} E_x^I \hat{y} e^{i(\omega_I t - k_I z)} \qquad (1)$$

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where E_x^I is a constant, ω_I is the circular frequency of the incident wave,

$$k_I = \frac{\omega_I}{c} = \omega_I\sqrt{\mu_0\epsilon_0},$$

$\eta = \sqrt{\mu_0/\epsilon_0}$, and μ_0, ϵ_0 are the free space permeability and permittivity, and $\hat{x}$, $\hat{y}$, $\hat{z}$ being the corresponding unit vectors.

The drifting gyrotropic plasma boundary, assumed to be located at $z = v_0 t$, will produce in general two reflected waves in free space

$$\bar{E}_{R1} = E_x^R \hat{x} e^{i(\omega_R t + k_R z)};$$
$$\bar{H}_{R1} = -\frac{1}{\eta} E_x^R \hat{y} e^{i(\omega_R t + k_R z)} \qquad (2a)$$
$$\bar{E}_{R2} = E_y^R \hat{y} e^{i(\omega_R t + k_R z)};$$
$$\bar{H}_{R2} = \frac{1}{\eta} E_y^R \hat{x} e^{i(\omega_R t + k_R t)} \qquad (2b)$$

where E_x^R, and E_y^R are constants, ω_R is the circular frequency of the reflected wave, and

$$k_R = \frac{\omega_R}{c} = \omega_R\sqrt{\mu_0\epsilon_0}.$$

In addition, one will have transmitted waves in the drifting magnetoplasma of the type $e^{i(\omega_T t - k_T z)}$. From Maxwell's equations one obtains for the plasma waves [2]

$$H_x = -\frac{k_T}{\mu_0\omega_T} E_y = -\frac{n}{\eta} E;$$
$$H_y = \frac{k_T}{\mu_0\omega_T} E_x = \frac{n}{\eta} E_x \qquad (3a)$$
$$\epsilon_0(n^2 - 1)E_x = sP_x;$$
$$\epsilon_0(n^2 - 1)E_y = sP_y \qquad (3b)$$

where $s = 1 - n\beta_L$, ω_T is the circular frequency of the transmitted electromagnetic plasma waves, k_T is the corresponding wavenumber, P_x, P_y are the space polarization components,

$$\beta_L = \frac{v_0}{c}, \qquad n = \frac{ck_T}{\omega_T} = \frac{c}{u_p},$$

is the refractive index, and u_p is the phase velocity of the wave in the drifting magnetoplasma. From the constitutive relations, one has for the plasma waves [3]

$$\epsilon_0 X_T E_x = -U_T P_x - iY_T P_y \qquad (4a)$$
$$\epsilon_0 X_T E_y = -U_T P_y + iY_T P_x \qquad (4b)$$

where the notation by Budden [4] has been used taking

$$X_T = \frac{\omega_p^2}{\omega_T^2}, \quad Y_T = \frac{\omega_{HL}}{\omega_T}, \quad Z_T = \frac{\nu}{\omega_T},$$
$$U_T = s - iZ_T = 1 - n\beta_L - iZ_T,$$

ω_p being the plasma frequency, ω_{HL} the longitudinal gyromagnetic frequency, and ν the collision frequency of the plasma. Substituting (3b) into (4), one obtains

$$[U_T(n^2-1) + sX_T]E_x = -iY_T(n^2-1)E_y \qquad (5a)$$
$$[U_T(n^2-1) + sX_T]E_y = +iY_T(n^2-1)E_x. \qquad (5b)$$

By equating the determinant of the homogeneous equations (5) to zero one may obtain for a nontrivial solution

$$(n^2 - 1)[1 - n\beta_L - iZ_T \mp Y_T] + (1 - n\beta_L)X_T = 0 \qquad (6)$$

where (6) is the well-known cubic refractive index equation [3] for the longitudinally drifting magnetoplasma, with respect to the circular frequency ω_T of the transmitted plasma waves. From (3) and by dividing (5) one may obtain the wave polarization of the plasma waves

$$R = \frac{E_y}{E_x} = \frac{P_y}{P_x} = -\frac{H_x}{H_y} = \mp i \quad (7)$$

where (7) is identical with the classical magneto-ionic theory for stationary plasma [4].

The effective values of the electric field $\bar{E}_{\text{eff}}$ and magnetic field $\bar{H}_{\text{eff}}$ of moving medium are given [5] by

$$\bar{E}_{\text{eff}} = \bar{E} + \bar{v}_0 \times \mu_0 \bar{H} \quad (8a)$$

$$\bar{H}_{\text{eff}} = \bar{H} - \bar{v}_0 \times \epsilon_0 \bar{E}. \quad (8b)$$

It has been shown [6] that the tangential boundary conditions at a moving boundary require

$$\bar{n} \times [\bar{E}_{\text{eff}}]_-^+ = 0, \quad \bar{n} \times [\bar{H}_{\text{eff}}]_-^+ = 0 \quad (9)$$

where there is no surface current density on the boundary and $[F]_-^+$ represents discontinuity $F^+ - F^-$ with $\pm$ signs relative to $\bar{n}$. Substituting (8) into (9) for the present case of no $\hat{z}$ field components, one has

$$[\hat{z} \times \bar{E} - v_0\mu_0\bar{H}]_{z=v_0t-} = [\hat{z} \times \bar{E} - v_0\mu_0\bar{H}]_{z=vt_0+} \quad (10a)$$

$$[\hat{z} \times \bar{H} + v_0\epsilon_0\bar{E}]_{z=v_0t-} = [\hat{z} \times \bar{H} + v_0\epsilon_0\bar{E}]_{z=vt_0+} \quad (10b)$$

where $z = v_0t-$ represents the free space side and $z = v_0t+$ represents the plasma side of the drifting boundary.

In order to obey the boundary conditions (10) at the drifting boundary, all waves will be required to have the same exponential time variation at $z = v_0t$, i.e.,

$$\omega_I t - k_I v_0 t = \omega_R t + k_R v_0 t = \omega_T t - k_T v_0 t. \quad (11a)$$

Using the above definitions one has the requirement

$$\omega_I(1 - \beta_L) = \omega_R(1 + \beta_L) = \omega_T(1 - n\beta_L) \quad (11b)$$

where $\beta_L = v_0/c$, and $n = c/u_p$. Assuming that the source frequency ω_I of the incident wave is given, one may find

$$\omega_R = \omega_I \frac{1 - \beta_L}{1 + \beta_L};$$

$$\omega_T = \omega_I \frac{1 - \beta_L}{1 - n\beta_L}. \quad (11c)$$

Because of the boundary conditions at the drifting boundary, the frequency of the reflected wave ω_R and of the transmitted wave in the plasma ω_T are given in terms of the frequency of the incident wave by (11c). Defining a new set of plasma parameters with respect to the circular frequency ω_I of the incident wave, denoting

$$X_I = \frac{\omega_p^2}{\omega_I^2}, \quad Y_I = \frac{\omega_{HL}}{\omega_I},$$

and

$$Z_I = \frac{\nu}{\omega_I}$$

and using together with (11c) in (6), one obtains

$$(1 - \beta_L)(n^2 - 1)[1 - \beta_L - iZ_I \mp Y_I] + X_I(1 - n\beta_L)^2 = 0. \quad (12)$$

Equation (12) is a quadratic equation for the refractive index $n = c/u_p$ of the drifting plasma, as compared to (6) which is a cubic equation. In general, one will have four different solutions for n in (12), two for $-Y_I$ and two for $+Y_I$; one of each pair will represent transmitted characteristic waves in the drifting plasma in the positive z direction and their refractive indices will be denoted by n_1 and n_2, respectively.

Using (3a) and (7) one has for the first transmitted plasma wave ($n = n_1$; $R = -i$)

$$\bar{E}_{T1} = E_x^{(1)}(\hat{x} - i\hat{y})e^{i(\omega_{T1}t - k_{T1}z)} \quad (13a)$$

$$\bar{H}_{T1} = i\frac{n_1}{\eta}E_x^{(1)}(\hat{x} - i\hat{y})e^{i(\omega_{T1}t - k_{T1}z)} \quad (13b)$$

where $E_x^{(1)}$ is a constant,

$$\omega_{T1} = \omega_I \frac{1 - \beta_L}{1 - n_1\beta_L}$$

and

$$k_{T1} = \frac{n_1\omega_{T1}}{c} = \frac{n_1\omega_I}{c}\frac{1 - \beta_L}{1 - n_1\beta_L}.$$

The corresponding second transmitted plasma wave ($n = n_2$, $R_2 = +i$) will be

$$\bar{E}_{T2} = E_x^{(2)}(\hat{x} + i\hat{y})e^{i(\omega_{T2}t - k_{T2}z)} \quad (14a)$$

$$\bar{H}_{T2} = -i\frac{n_2}{\eta}E_x^{(2)}(\hat{x} + i\hat{y})e^{i(\omega_{T2}t - k_{T2}z)} \quad (14b)$$

where $E_x^{(2)}$ is a constant,

$$\omega_{T2} = \omega_I \frac{1 - \beta_L}{1 - n_2\beta_L}$$

and

$$k_{T2} = \frac{n_2\omega_{T2}}{c} = \frac{n_2\omega_I}{c}\frac{1 - \beta_L}{1 - n_2\beta_L}.$$

Both waves are circularly polarized but in opposite directions; because of the boundary conditions at the drifting boundary, the waves will propagate with different circular frequencies ω_{T1}, ω_{T2} in the plasma for the same frequency ω_I of the incident wave.

Assuming that the amplitude E_x^I of the linearly polarized incident plane wave in (1) is known, one can find E_x^R, E_y^R of the reflected waves in (2) and $E_x^{(1)}$, $E_x^{(2)}$ of the transmitted waves in (13) and (14) by using the four boundary conditions in (10) at the drifting boundary $z = v_0t$. Substituting (1), (2), (13), and (14) in the boundary conditions (10), cancelling the identical exponential time variations at $z = v_0t$, taking

$$v_0\epsilon_0\eta = \frac{v_0\mu_0}{\eta} = \frac{v_0}{c} = \beta_L$$

and rearranging, one obtains

$$\begin{bmatrix} -(1+\beta_L) & 0 & +(1-n_1\beta_L) & +(1-n_2\beta_L) \\ 0 & -i(1+\beta_L) & +(1-n_1\beta_L) & -(1-n_2\beta_L) \\ 0 & +i(1+\beta_L) & +(n_1-\beta_L) & -(n_2-\beta_L) \\ +(1+\beta_L) & 0 & +(n_1-\beta_L) & +(n_2-\beta_L) \end{bmatrix} \begin{bmatrix} E_x^R \\ E_y^R \\ E_x^{(1)} \\ E_x^{(2)} \end{bmatrix} = \begin{bmatrix} 1-\beta_L \\ 0 \\ 0 \\ 1-\beta_L \end{bmatrix} E_x^I \quad (15)$$

where (15) is identical with the result found by Chawla and Unz [1] for the relativistic case. Once n_1 and n_2 for the transmitted plasma waves are found from (12), one may find reflected waves E_x^R, E_y^R and the drifting plasma transmitted waves $E_x^{(1)}$, $E_x^{(2)}$ in terms of the incident wave E_x^I after multiplying (15) by the inverse of the square matrix.

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HILLEL UNZ
Dept. Elec. Engrg.
University of Kansas
Lawrence, Kans. 66044

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An Experimental Gas Lens Optical Transmission Line

The advent of successful gas lenses [1]–[4] started investigations of their performance in optical transmission lines. A test line of spaced thermal gas lenses was built at Holmdel in a long corridor using parts and techniques available from previous circular waveguide work.

The gas lens elements of this line consisted of copper tubes 15 cm long with an inside diameter of 6.3 mm and an outside diameter of 9.5 mm. They were heated by a single layer winding of number 34 enameled resistance wire having a total resistance of 1400 ohms. A thermocouple was soldered to the center of each lens tube to determine its temperature.

Each lens element was foamed in place in the center of a one-half meter length of 2-inch

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